



## TSG 27 Learning and cognition in mathematics

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### Focus

Learning and cognition is a classical and very vital area in research on mathematics education. Different to many other special and related TSGs, such as teaching and learning of algebra, geometry, measurement, statistics, calculus, to mention a few, this TSG has a more general focus.

Originally, research was focused mostly on the cognitive processes taking place in the individual. The past twenty-five to thirty years, however, the research has expanded. Research on learning as well as mathematical cognition are now frequently framed with socio-cultural theories, and closer connections are being made between social and cognitive theories. In addition, influences of materials, classroom contexts, and affective factors such as emotions, beliefs, and attitudes on learning and cognition are foci of interest.

This TSG on learning and cognition in mathematics covers any teaching and learning contexts in which the learning of mathematics is found to occur (e.g., early childhood to tertiary level, adult education, pre-service teacher education, teacher professional development, and also learning and cognition outside the school context). The range of foci is broad, and the aim is to gather research on the characterization of mathematical learning processes, cognitive processes associated with mathematical thinking and problem solving, learning as an individual or as a social activity, learning contexts (including intercultural contexts), the impact of task design on learning and cognition, and how individual differences and learner characteristics can affect learning processes.

We welcome contributions in the broad domains of research described above, and specific contributions that relate to new and ongoing trends, of which we mention a few:

1) Research into the early roots of mathematical cognition, as well as the ways in which this translates in mathematical knowledge at a later age. It is more and more acknowledged in research that children at a very young age represent and process numerical magnitudes, and – as shown by longitudinal research – these magnitude representations are highly predictive for later mathematical achievement.

2) Early roots of mathematical cognition are not necessarily restricted to numerical magnitude understanding. Current research also takes interest in children's quantitative reasoning skills as an important precursor for mathematical development. Similarly, research investigates the role of individual differences in children's spontaneous attentional processes with regard to for example, number and quantitative relations.

3) Mathematical learning is not necessarily considered as a continuous and smooth process of development of mathematical cognition wherein learners' prior knowledge is always useful to develop new concepts. The conceptual change approach assumes that in certain cases, learners' prior knowledge (developed outside as well as inside formal mathematics education) sometimes needs to be considerably revised to gain understanding of a new mathematical concept. From a different theoretical perspective, categorization of parts of the Zone of Proximal Development (ZPD) has provided a tool to answer questions about why children sometimes revert to simple mathematical procedures when they have demonstrated proficiency with more complex and appropriate procedures.

4) It is more and more acknowledged that mathematical cognition is not or not always purely rational in nature. Intuitions or heuristics play an important role in mathematical thinking, and may cause a learner to arrive at solutions to problems in a very efficient way without intensive processing. But the strong impact of intuitions may also cause the learner to make mistakes in situations where he or she in principle possesses all the required knowledge and skills to arrive at the correct solution.

5) Closer and closer connections continue to be made between cognitive, social, and affective and psychological processes associated with mathematics learning, and the part that cognition, social interactions, and materials available play in the learning process. Research on embodied cognition, and research on the flow of ideas (in both directions) between individuals, groups, and the whole class provide have been shown to be productive directions.

6) Recent links made between spontaneous student mathematical activity, the role that certain types of modelling tasks can play in enabling spontaneity, and eliciting engagement are present areas of international research focus that

fit well with the current international interest in STEM education (Science, Technology, Engineering, and Mathematics Education).

## **Programming**

Participant thinking will be stimulated at the start of each of the four 90 minute sessions by an invited keynote speaker, who addresses the topic from a specific perspective. Each keynote speaker will first be asked to address how he or she defines mathematical cognition and / or mathematics learning.

In the first session, Erno Lehtinen provides a keynote lecture, with a focus on mathematical cognition and learning processes as situated within the individual learner.

In the second session, Dor Abrahamson will focus on the emergence of mathematical understanding as an embodied and socioculturally situated process.

In the third session, Minoru Othani gives a keynote lecture on specific ways in which mathematical cognition can be stimulated in a classroom setting: which aspects of learning environments (e.g., task designs, task implementation, lesson structure) are conducive to this, as well as how mathematical understandings are developed in classroom contexts.

In the fourth session, Judy Anderson gives a keynote lecture on characteristics of the individual learner that may affect their engagement with mathematics. Personal, psychological, and / or emotional factors, as well as affective and creative processes that can influence how mathematical learning takes place will be explored, as well as how teaching can stimulate creative learning processes.

Abstracts of the keynote speaker are provided below.

## **Call for contributions**

We invite contributions that fit within the TSG theme as described above. In each session, besides the time devoted to the invited keynote speaker, time will be allotted to a limited number of oral presentations that *in some way* relate to the keynote presentation.

## **Abstracts**

### ***Erno Lehtinen***

#### ***Mathematical cognition and learning processes: Analysis of individual learning trajectories***

In the presentation I will focus on the development and extensions of number concept before formal schooling and during primary school. I will show how individual level cognitive analysis help us to understand learning trajectories,

crucial changes, and different processes leading to deep and flexible understanding of number systems in some individuals, and superficial and inflexible number knowledge in others. On the basis of our own research I will describe inter-individual differences in children's own spontaneous quantitative focusing tendencies which partly explain the differences in the way how children learn to understand natural numbers and which prepare students for conceptual changes needed in learning rational numbers. At the end of the presentation, I will discuss about educational consequences of these findings.

### ***Dor Abrahamson***

This lecture will survey key idea from the recent "E-turn" in mathematics education, namely perspectives from cognitive science that conceptualize the mind as embodied, extended, and enactive activity in natural and sociocultural ecologies. We will argue for these views, claiming that educational researchers investigating instructional interactions still need intellectual and methodological frameworks for conceptualizing, designing, facilitating, and analyzing how students' immersive hands-on dynamical experiences become formulated within semiotic registers typical of mathematical discourse. We present analyses of integrated videography, action logging, and eye-tracking data from tutor–student clinical interviews using a technologically enabled embodied-interaction learning environment, the Mathematical Imagery Trainer for Proportion, to describe the emergence of mathematical concepts from the guided discovery of sensorimotor schemes.

Central to the presentation is the theoretical construct of an attentional anchor. Borrowed from radical-enactivist and ecological-dynamics perspectives on sports cognition and performance, attentional anchors are environmental elements, features, or aspects that mediate teaching, learning, and enactment of motor-action skill. Our empirical data suggest that students are inventing attentional anchors as their spontaneous solution to a challenging bimanual-coordination interaction problem. These attentional anchors then become objectified via the uptake of available frames of reference so as to serve in the mediation of mathematical visualizations and concepts. As such, Abrahamson reconciles constructivist and sociocultural models by underscoring the role of artifacts and facilitation in the micro-events of mathematical ontogenesis. In particular, technologies geared both to foster and monitor embodied mathematics learning may renew the field's interest in Piaget's reflective abstraction.

***Minoru Ohtani***

To be added

***Judy Anderson***

***The Middle Years Transition, Engagement and Achievement in Mathematics (MYTEAM) Project***

Prompted by international concerns about school and post-school participation in mathematics, the MYTEAM project sought to investigate factors affecting student engagement in the Australian context. With an initial sample of 4383 students in 257 classrooms and 47 schools, five studies investigated: cross-sectional construct validity of several instruments measuring motivation and engagement, attitude, classroom environment and achievement; transition and causal modeling across grades 5-8; longitudinal study examining 'switching on' and 'switching off'; interviews with teachers and students as well as observations of classrooms with high levels of student engagement to identify specific pedagogies; and a 10-week intervention study in one school. Based on longitudinal matched data from 1601 students, multilevel regression modeling findings showed that compared with grade 6 students, those in grades 7 and 8 significantly declined in mathematics engagement from the previous year. Also, variance component models showed the bulk of the variance (>85%) in future intent and disengagement resided at the student level, with 10-12% variance explained at classroom and school levels. For future intent and disengagement, mathematics self-efficacy, valuing, enjoyment, perceived classroom enjoyment, and parent interest were significant predictors. For disengagement, additional predictors were mathematics anxiety, perceived classroom disengagement, school ethnic composition, and school socioeconomic status. Teacher interviews and observations revealed a range of practices to engage students and highly engaged students were more 'alike' in their attitudes, strategies for learning, interest and behavior regardless of achievement levels. In the intervention, teachers' responses to engagement were mediated by personal and contextual elements including efficacy beliefs, confidence in mathematics and conceptions of student engagement.