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# Knowledge in Pieces: A Framework for Studying Learning at High Resolution

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Knowledge in Pieces (KiP) is an epistemological framework—it focuses on the very nature of knowledge—which has flourished mainly in science education over the last 25 years. This talk aims to communicate those ideas to mathematics educators, and to show how the framework can apply to learning mathematics. A distinctive feature of KiP is that it deals with deep and difficult learning *in the details of learning-in-process*. It allows tracking learning at "high resolution," using real-time data to uncover learning steps at the grain-size at which learners experience it, and at which teachers and curriculum developers try to manage it. This rare kind of analysis can be unusually helpful in designing instruction and learning materials.

I first characterize the overall KiP program of studies and contrast it with other programs of understanding learning. Then, I use recent studies to illustrate KiP principles in action, reflecting back on the global differences KiP exhibits compared to other frameworks. The sample studies aim at a balance, showing KiP in its original and most successful form (in science) while still providing a good bridge to the case of mathematics. They also emphasize how KiP enfolds both the broad trajectory of long-term learning, but also in-the-moment insights by learners.

Sample studies include:

1. During a classroom episode, students develop, on their own, a normative mathematical view of an important physical phenomenon: temperature equilibration. One can see here, element by element, what incoming intuitive knowledge was invoked, and how it changed and combined to result in the normative idea.
2. (briefly) This study involves micro-analysis of student learning from a well-known instructional sequence in science. In this case, we can track differences in incoming student knowledge well enough to see why the same instruction led sometimes to success and sometimes to failure.

The second pair of studies emphasizes longer-term learning:

3. A study shows how learning is *not* monotonic (always-increasing number of "correct" elements) and how changing contextuality (when students use particular ideas) constitutes an important dimension of learning.
4. (briefly) A long-term study of mathematics learning (statistics) shows how progress can consist of: (a) invoking numerous naïve knowledge elements in different contexts, and (b) gradually expanding use of learned ideas.