

**TWO-AXIS PROCESS MODEL FOR TEACHING AND LEARNING OF SCHOOL
MATHEMATICS**

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Based on theoretical studies to identify basic components for a descriptive and prescriptive model of mathematical understanding, the author developed the two-axis process model. The model consists of two axes, the vertical and the horizontal. The vertical axis comprises three levels of understanding mathematics; (V1) mathematical entities, (V2) a relation of the entities, and (V3) the general relation. The horizontal axis comprises three stages of learning at each level; (H1) intuitive stage (Students are provided opportunities for manipulating concrete objects, or operating on mathematical concepts and relations acquired in a previous level. At this stage, they do intuitive thinking.), (H2) reflective stage (Students are stimulated and encouraged to pay attention to their own manipulating or operating activities, to be aware of them and their consequences, and to represent them in terms of diagrams, figures or language. At this stage, they do reflective thinking.), (H3) analytic stage (Students elaborate their representations to be mathematical ones using mathematical terms, verify the consequences by means of other examples or cases, or analyze the relations among consequences in order to integrate them as a whole. At this stage, they do analytic thinking.).

As a result of theoretical and practical studies, the author identified three principles (P1, P2, P3) and three methods (M1, M2, M3) for designing mathematics lessons based on the model to facilitate and deepen students' mathematical understanding; (P1) recognizing mathematical understanding as a dynamic process, (P2) setting up levels of understanding and learning stages at a level, (P3) incorporating students' individual constructions and social constructions; (M1) making clear levels of understanding related to a certain mathematical topic, (M2) assessing and evaluating students' understanding as a readiness, (M3) planning in detail three learning stages as a dialectic process of individual and social constructions in a lesson.

In the invited lecture, the author will demonstrate that the two-axis process model with the principles and methods can be used by teachers as an effective general framework for designing mathematics lessons to improve their students' mathematical understanding in a classroom. For that purpose, we look at some case studies on the students' process of understanding in primary and secondary school mathematics in Japan. Finally the author will summarize some suggestions for developing the teaching and learning of school mathematics by using the two-axis process model as a general framework for designing mathematics lessons.