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LEARNING MODERN ALGEBRA FROM EARLY ATTEMPTS TO PROVE FERMAT'S LAST THEOREM: A COURSE FOR PROSPECTIVE TEACHERS

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Modern abstract algebra finds applications in many mathematical professions. This talk concerns its applications to the profession of mathematics teaching at the secondary level (ages 14—18, "high school" in the US). Major themes in modern algebra—structure, extension, decomposition, reduction, localization, and representation, among others—can help teachers bring coherence and parsimony to the entire high school curriculum. For example, the common (and often discouraged) impulse of high school students, thinking about arithmetic in complex numbers as ``algebra with *i*" with an extra simplification rule, has quite a solid pedigree in modern algebra and can provide a glimpse of the reduction technique used to construct splitting fields for polynomials. Another example: In polynomial algebra, being explicit about the interplay between formal and functional thinking (something that is often blurred in secondary texts) helps students develop an appreciation for the "two faces" of algebra (Weyl, 1995). But abstract algebra provides more than a deeper perspective on school mathematics; it also provides teachers with tools that can be applied to aspects of their work that don't necessarily end up in the hands of students. One can use the arithmetic of certain quadratic fields to construct tasks for students that have integer or rational solutions—for example a general method for constructing scalene triangles with integer side lengths, one of whose angles measures 60 degrees. This is genuine applied algebra—algebra applied to the profession of teaching.

These profession-specific applications of algebra are often missing in courses for prospective teachers, and many teachers I know see their abstract algebra courses as interesting but disjoint from their teaching practice. Joseph Rotman and I have designed a course that highlights the applications of modern algebra to teaching (Cuoco and Rotman, 2013). The goal of making explicit connections to the secondary curriculum for an audience of pre-service high school teachers led to several design principles that will be detailed in the talk. Three of the most important are to emphasize rings and fields over groups (CBMS, 2012) (in the spirit of the classic text of Birkhoff and Mac Lane (1977)), to organize the material with a focus on how its evolution was tied to early attempts to prove Fermat's Last Theorem, and to show how the major systems of pre-college mathematics—the ordinary integers and polynomials in one variable over a field—share structural similarities that, put in the proper abstract setting, are present in many other algebraic systems.

References

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