MATHEMATICAL PROBLEM SOLVING IN CHOICE-AFFLUENT ENVIRONMENTS

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Keen interest in problem solving emerged in the mathematics education community many decades ago and has been sustained until today for a clear reason: mathematical problem solving (including problem posing, conjecturing, and proving) is the central activity in mathematics as a living science, and thus it has been hoped that it would also become the central activity in mathematics education. Somewhat surprisingly, this expectation is not yet fulfilled, despite a great deal of knowledge accumulated on how problem solving occurs and what can be learned through problem solving (Mamona-Downs & Downs, 2005; Schoenfeld, 1992, 2013). Many fundamental queries related to the role of problem solving in mathematics education have proven to be hard nuts to crack. Examples of such worthy queries are: (i) What are the key characteristics of learning environments in which school students have a fair chance to invent (rather than recall or be told) solution ideas to challenging problems? (ii) How can such environments be designed, supported, and sustained in real-life instructional settings?

In order to tackle these queries, I first present a framework in which the process of inventing a solution is considered as the solver's pathway of delicate shifts of attention (this term is due to Mason, 1998). The framework embraces three types of sources that can stipulate one's pathway of attention shifts: individual effort and resources; interaction with peer solvers who struggle in their own ways with the problem, or attempt to solve it together; and interaction with a source of knowledge about the solution, such as a teacher or a textbook. The framework is implemented to analyze two data sets. The first data set is from a study in which middle school students solve, sometimes for several days, geometry problems in a learning environment combining classroom and on-line activities. The second data set is from a study in which middle-school students solve, sometimes for several weeks, challenging algebraic problems in a project-based learning environment. The exposition converges to the following conclusion: successful cases of student problem solving are more likely to occur in choice-affluent learning environments, namely environments in which the students are empowered to make informed choices of: a challenge to be dealt with, a way of dealing with the challenge, a mode of interaction, an extent of collaboration, and an agent to learn from. Some thoughts about how to construct choice-affluent situations in school reality are presented.

References

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